

# Thermalization in heavy ion collisions from holography

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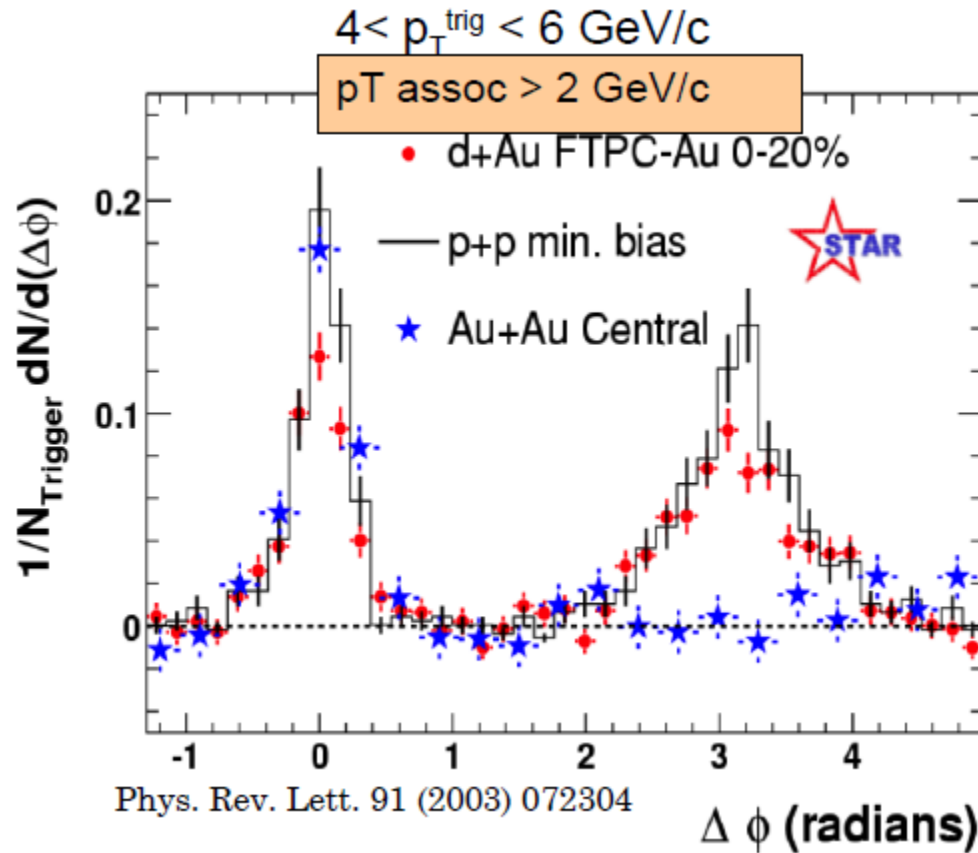


HENPIC, March 13 2014

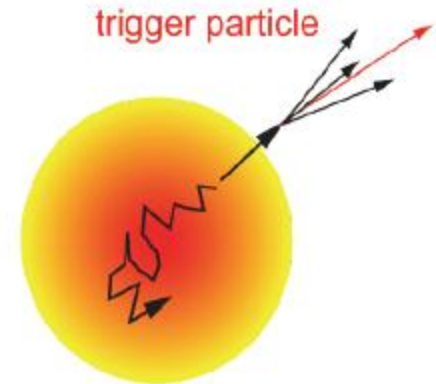
# Outline

- Introduction to thermalization problem.
- The theoretical tool: AdS/CFT.
- A gravity dual of heavy ion collisions and hologram on the boundary.
- Gravitational collapse model and thermalization
- Two point function as probe of thermalization: top-down thermalization scenario?
- Quasi-static approximation and geometric optics approximation
- Summary

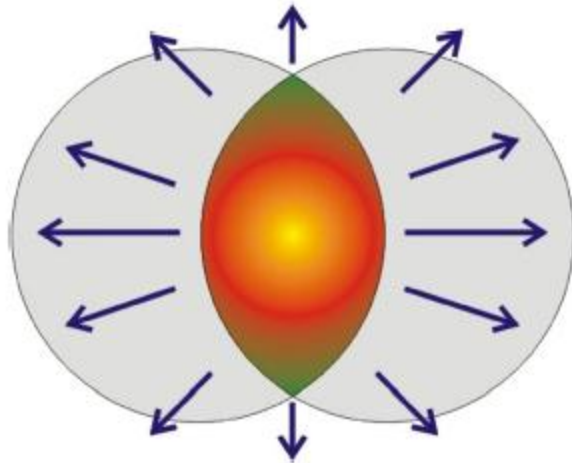
# Evidence for Quark Gluon Plasma (QGP)



Jet quenching



# Evidence for QGP



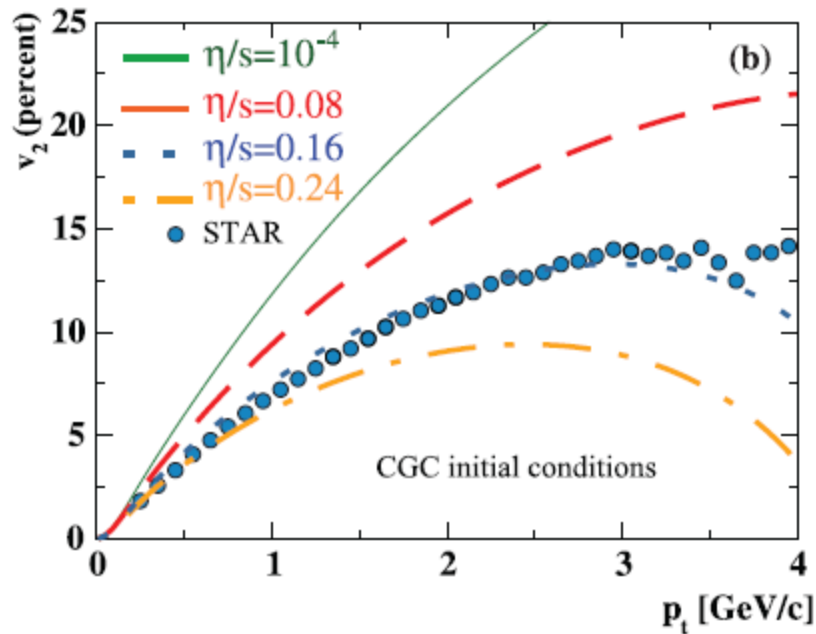
$$E \frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi} \frac{d^2N}{p_t dp_t dy} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_{\text{RP}})] \right)$$

$\varphi$  the azimuthal angle,

$\Psi_{\text{RP}}$  the reaction plane angle.

Initial spatial anisotropy → azimuthal anisotropy in particle production

# Hydrodynamic description of Elliptic flow v2



$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

$$\tau^{\mu\nu} = -\eta P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \left(\zeta - \frac{2}{3}\eta\right) P^{\mu\nu} \partial \cdot u,$$

$\eta$ : shear viscosity

$\zeta$ : bulk viscosity

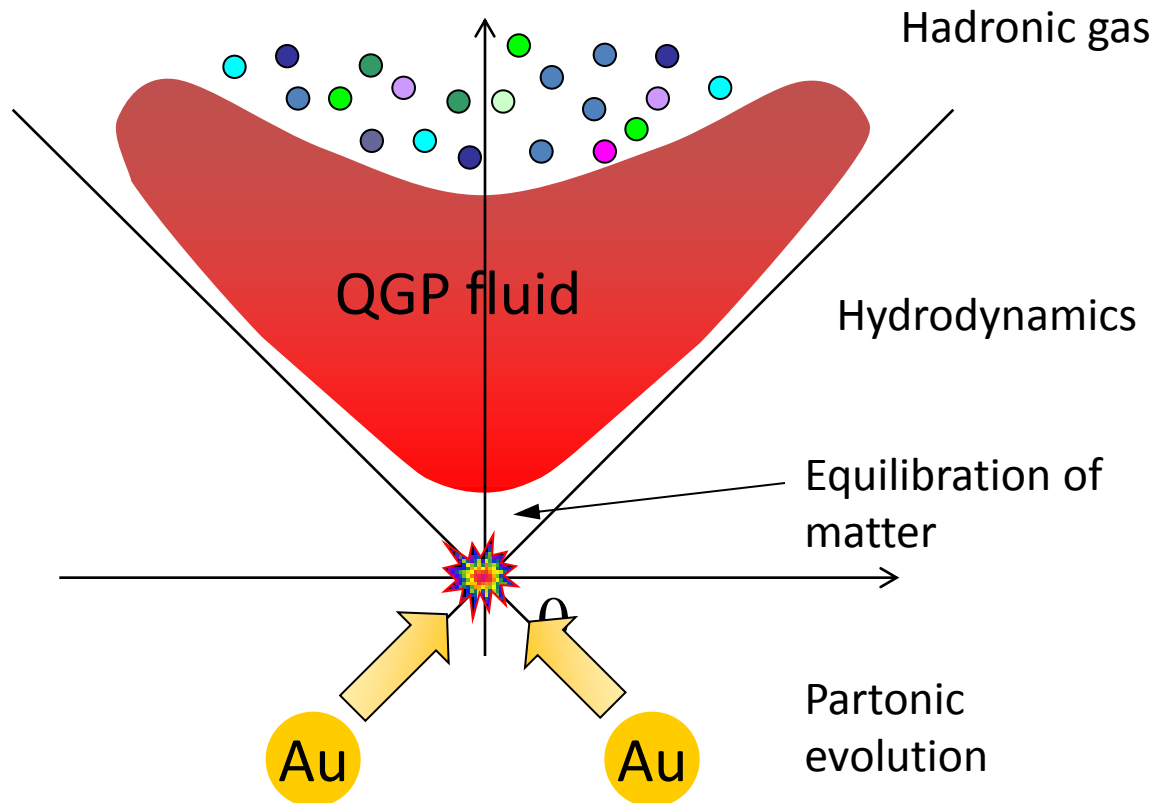
Hydrodynamics requires field  
reaches local thermal equilibrium

One input parameter of  
hydrodynamics:

Onset time of hydrodynamics:

Thermalization time  $\sim 0.5$  fm/c

# Stages of heavy ion collisions



thermalization  
time scale  $0.5\text{fm}/c$   
 $\Rightarrow$   
Fast thermalization,  
suggests strong  
interaction (QCD at  
strong coupling regime)

# AdS/CFT correspondence

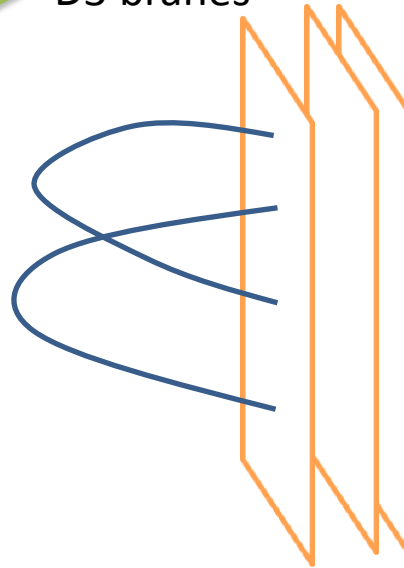
Large  $N_c$ , strong coupling  $\lambda$  limit of  $N=4$  SYM



Classical SUGRA in AdS background

low energy theory of  $N_c$   
D3 branes

3+1D worldvolume fields:  
SU(N) gluons etc



D3 brane can be viewed as  
soliton in 10D, alternatively  
described by SUGRA fields:  
graviton, dilaton etc

# AdS/CFT preliminaries

Pure AdS

$$ds^2 = \frac{L^2}{z^2} (-dt^2 + d\vec{x}^2 + dz^2)$$

N=4 SYM at zero temperature  
(vacuum)

AdS-  
Schwarzschild

$$ds^2 = \frac{L^2}{z^2} (-f(z)dt^2 + d\vec{x}^2 + dz^2 / f(z))$$

$$f = 1 - \frac{z^4}{z_h^4}$$

N=4 SYM at temperature  
(plasma)

$$T = \frac{1}{\pi z_h}$$

bulk field



boundary operator

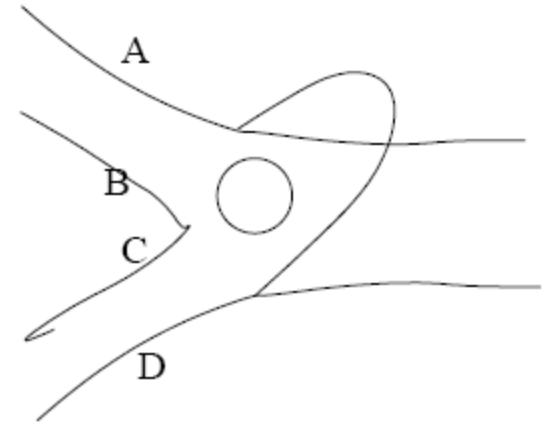
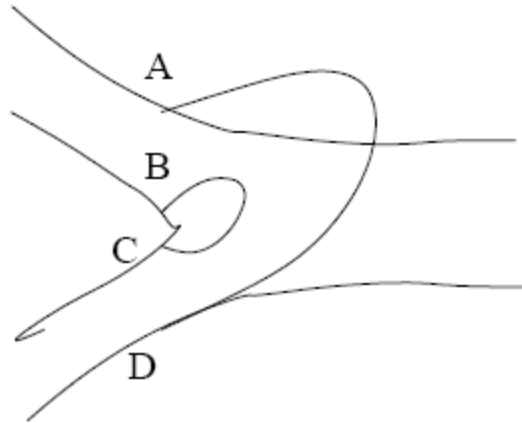
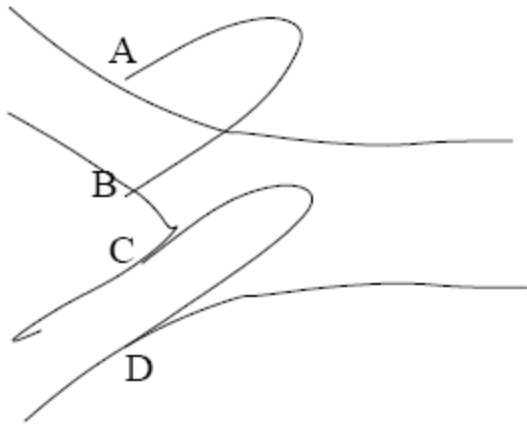
dictionary

$\phi$   
 $A_\mu$   
 $h_{\mu\nu}$

$O = \text{Tr} F^2 + \dots$   
 $J^\mu$   
 $T_{\mu\nu}$

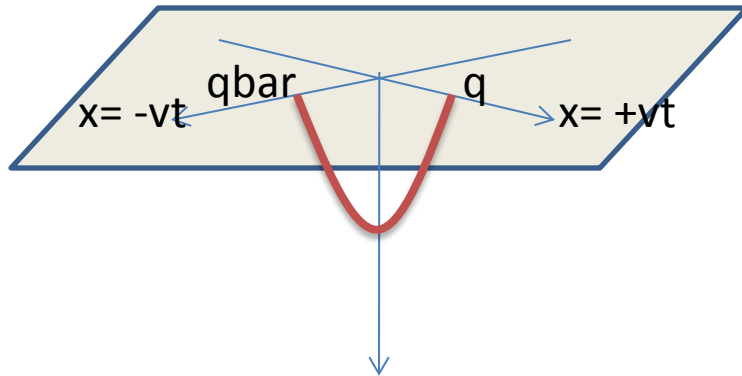


# A gravity dual of heavy ion collisions



Example: a collision of two open string (mesons) results in one closed string (debris) and one open string (separating quark/anti-quark)

# Scaling string



separating quark/anti-quark

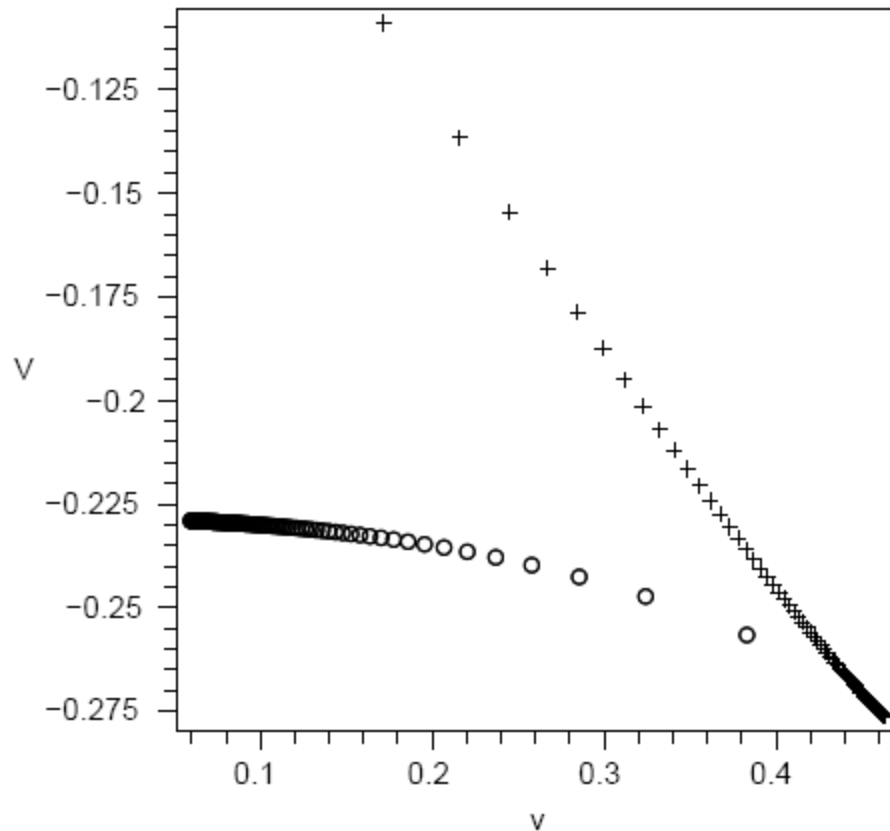
Scaling solution exists for  $v < 0.462$

$$S = -\frac{R^2}{2\pi\alpha'} \int \frac{\tau d\tau dy}{z^2} \sqrt{1 - \left(\frac{\partial z}{\partial \tau}\right)^2 + \frac{\left(\frac{\partial z}{\partial y}\right)^2}{\tau^2}}$$

$$z(\tau, y) = \frac{\tau}{f(y)}$$

$$y = f_0 \sqrt{\frac{f_0^2 - 1}{2f_0^2 - 1}} F \left( \sqrt{\frac{f^2 - f_0^2}{f^2 - 1}}, \frac{f_0}{\sqrt{2f_0^2 - 1}} \right) - \frac{1}{f_0} \sqrt{\frac{(f_0^2 - 1)^3}{(2f_0^2 - 1)}} \Pi \left( \sqrt{\frac{f^2 - f_0^2}{f^2 - 1}}, \frac{1}{f_0^2}, \frac{f_0}{\sqrt{2f_0^2 - 1}} \right)$$

# v-dependent potential



Ampere's law at strong coupling

$$V = 0.2285 \frac{(1 + 0.6830 v^2) \sqrt{g^2 N}}{L}$$

# Hologram of scaling string

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa^2 T_{\mu\nu}$$

Stress energy tensor on the boundary CFT:  $T_{mn} = 2/\kappa^2 * Q_{mn}$

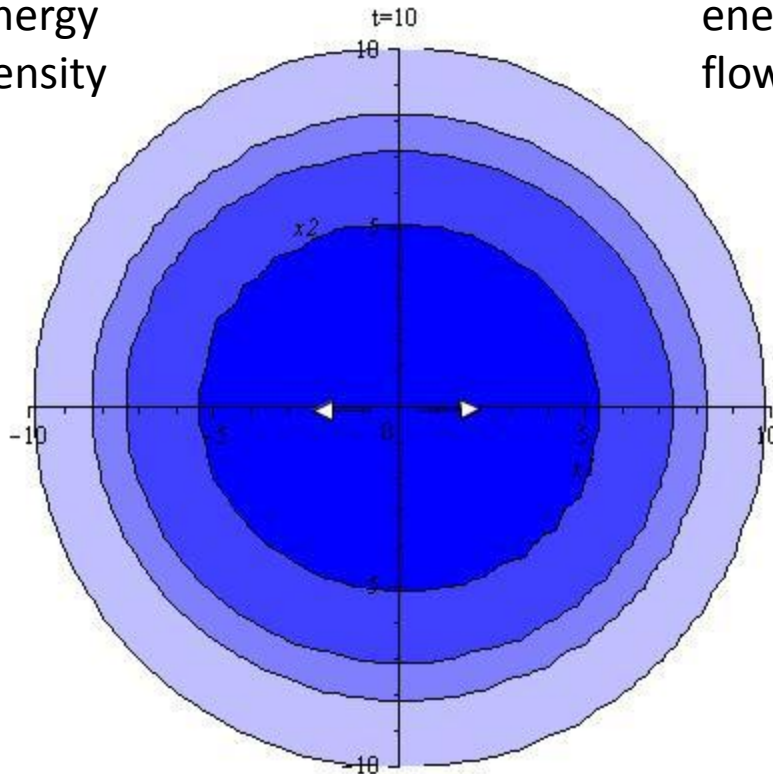
$$Q_{mn}(t', x') = \int P_R(t' - t, x' - x, z) s_{mn}(z, t, x) dz dt d^3x$$

$$P_R = \frac{12iz}{(2\pi)^2} \left[ \frac{1}{(t^2 - r^2 - z^2 + i\epsilon)^4} - \frac{1}{(t^2 - r^2 - z^2 - i\epsilon)^4} \right] \theta(t - r)$$

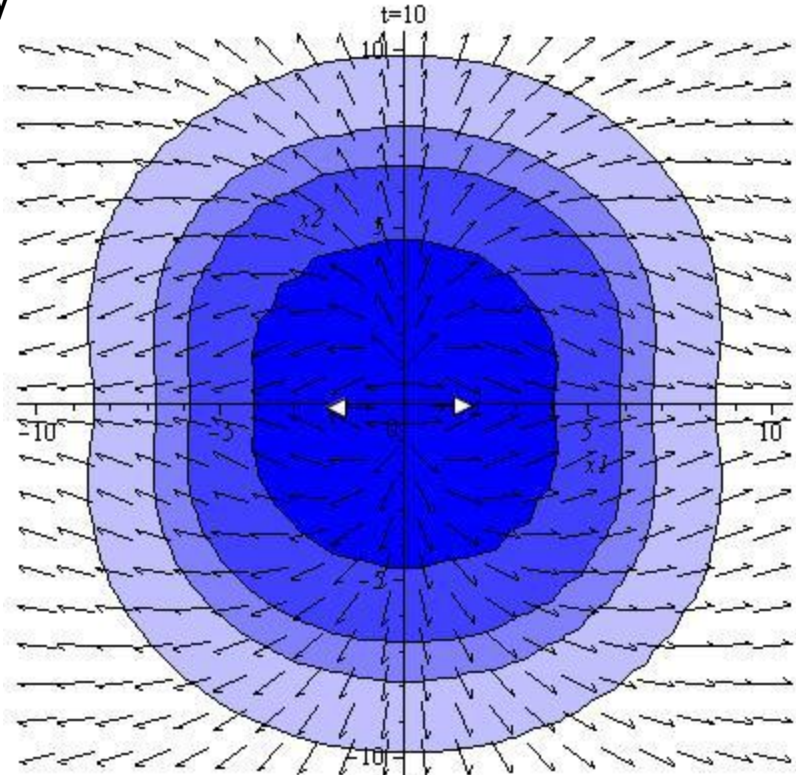
Propogator:  $P_R$   
 Source:  $s_{mn}$  linear  
 combination of  $T_{\mu\nu}$

# Hologram of scaling string

energy  
density



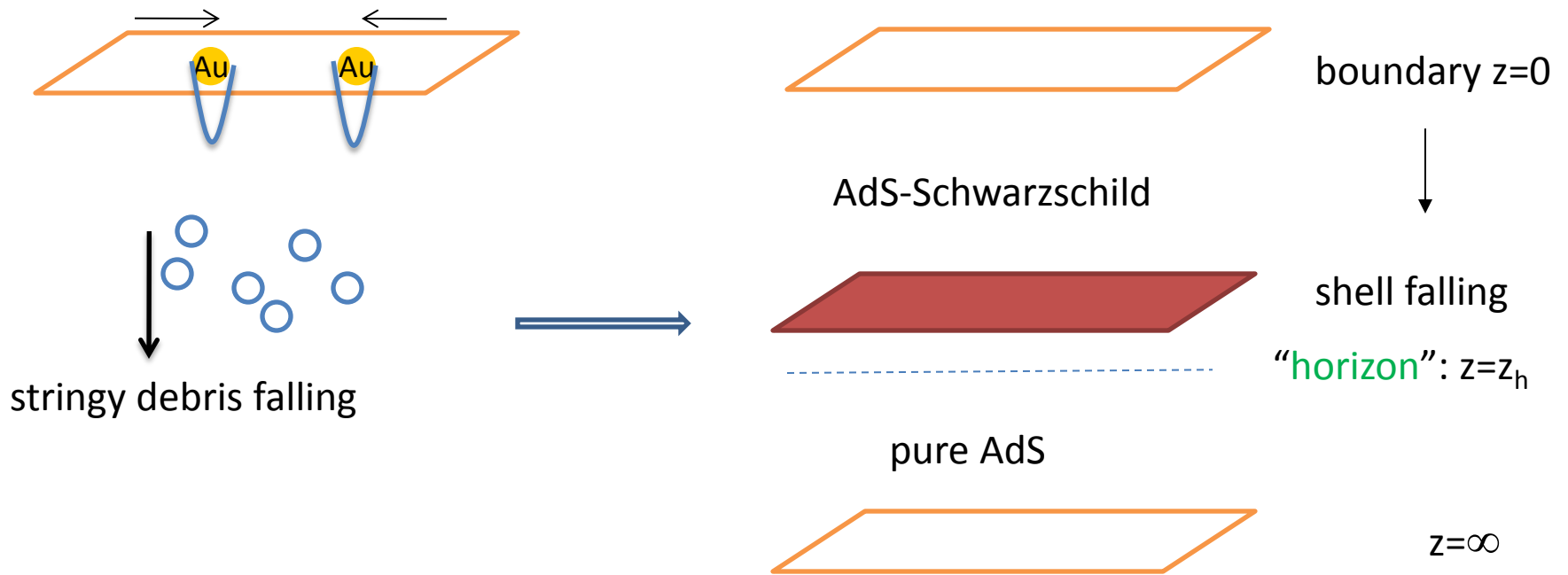
energy  
flow



No jet like structure! Nearly isotropic distribution.  
quark anti-quark pair remain strongly correlated all  
the time.

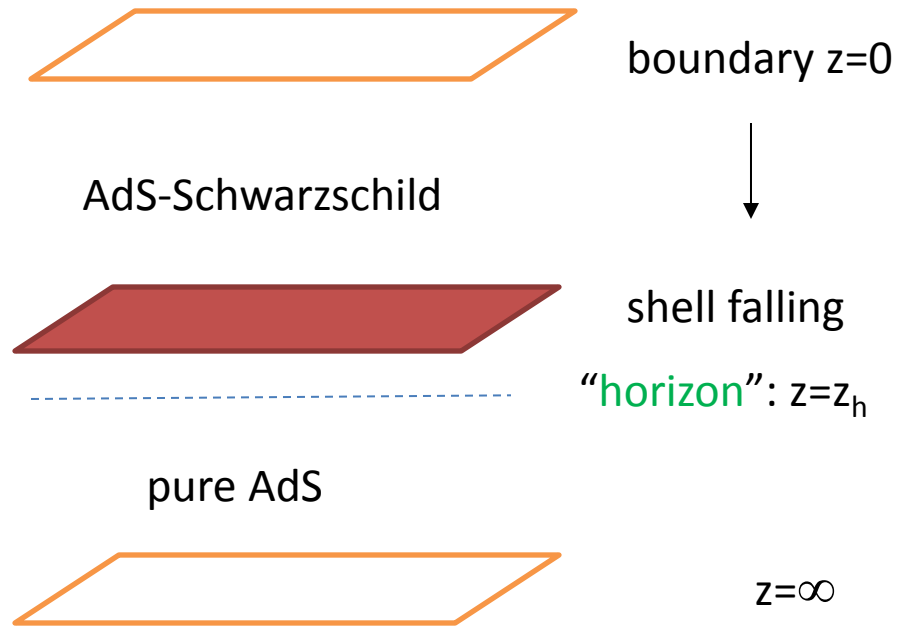
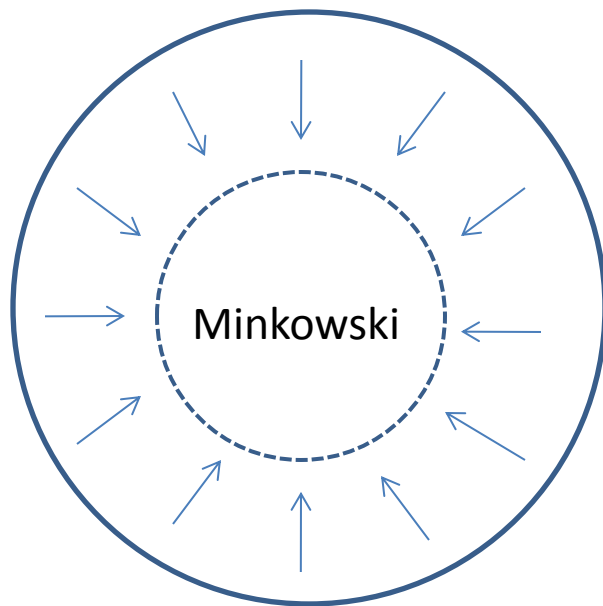
SL, Shuryak PRD 07

# Gravitational collapse dual to thermalization



# Gravitational Collapse in AdS & flat space

Schwarzschild



# Trajectory of falling shell from Israel junction condition

Israel junction condition

$$[K_{ij} - \gamma_{ij}K] = \kappa S_{ij}, \quad \{K_{ij}\}S^{ij} = 0.$$

$K_{ij}$ : extrinsic curvature

$\gamma_{ij}$ : intrinsic curvature

$$S_{ij} = (\epsilon(z) + p(z))u_i u_j + p(z)\gamma_{ij}. \quad \text{Equation of State} \quad \epsilon = \frac{p}{a}$$

$$\Rightarrow \dot{z} = \sqrt{\frac{1}{4} \left( bz^4 + \frac{1}{bz_h^4} \right)^2 - 1}, \quad \dot{t}_f = \frac{\sqrt{f + \dot{z}^2}}{f} = \frac{\frac{1}{bz_h^4} - bz^4}{2f}.$$

$$bz_s^4 + \frac{1}{bz_h^4} = 2.$$

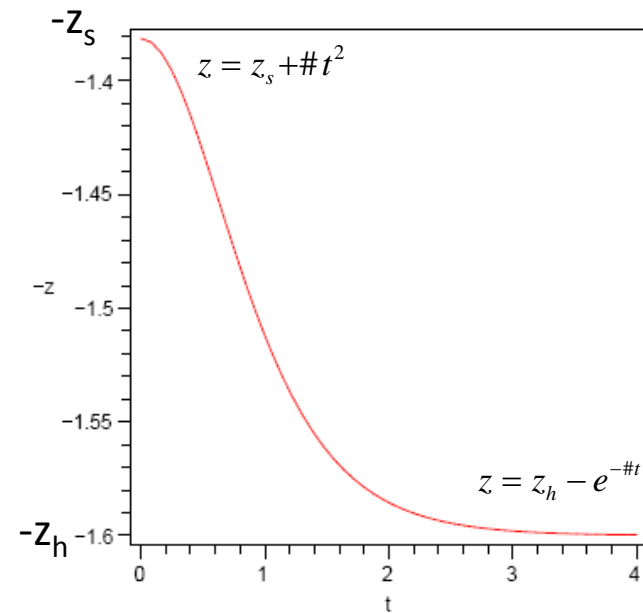
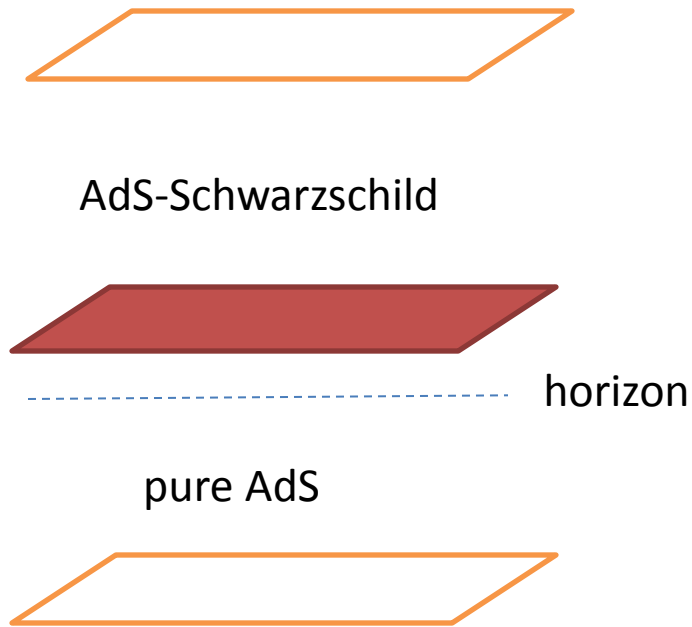
$z_s$ : initial shell position (intrinsic scale)

$b$ : “energy density”

$z_h$ : horizon position (temperature)



# Trajectory of falling shell from Israel junction condition



# One-point function vs Two-point function

$$T_{\mu\nu} = \text{diag}(\varepsilon, p, p, p) = T_{\mu\nu}^{\text{thermal}}$$

One-point function same as thermal counterpart:  
thermalizes instantaneously

$$\langle T^{\mu\nu}(x) T^{\lambda\rho}(y) \rangle \neq \langle T^{\mu\nu}(x) T^{\lambda\rho}(y) \rangle^{\text{thermal}}$$

$$\langle O(x) O(y) \rangle \neq \langle O(x) O(y) \rangle^{\text{thermal}}$$

...

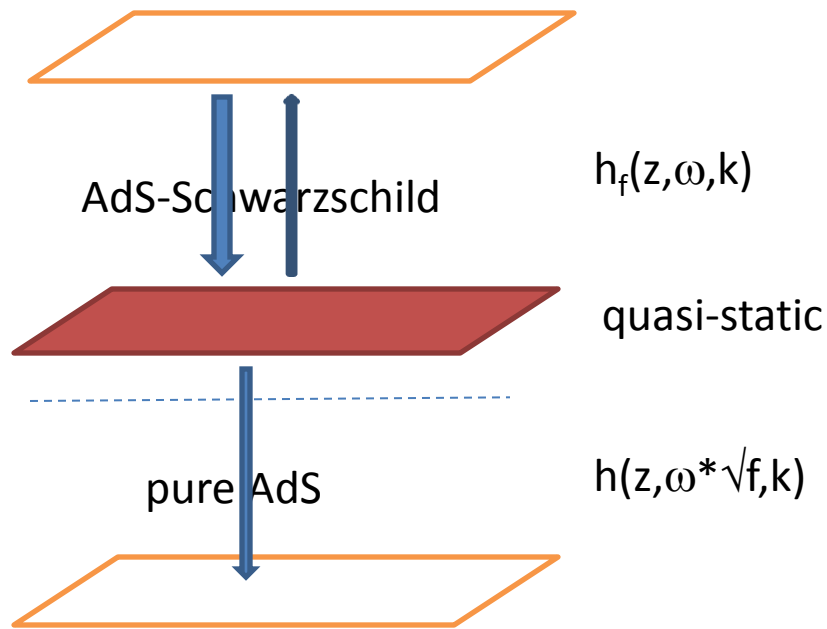
Focus on



Two-point function in general differs from thermal  
counterpart, measures approach to equilibrium

Similar model (Chesler, Teaney 11) also indicates two-point function takes  
longer time to thermalizes than one-point function

# Correlator for stress tensor from bulk graviton in **quasi-static** approximation



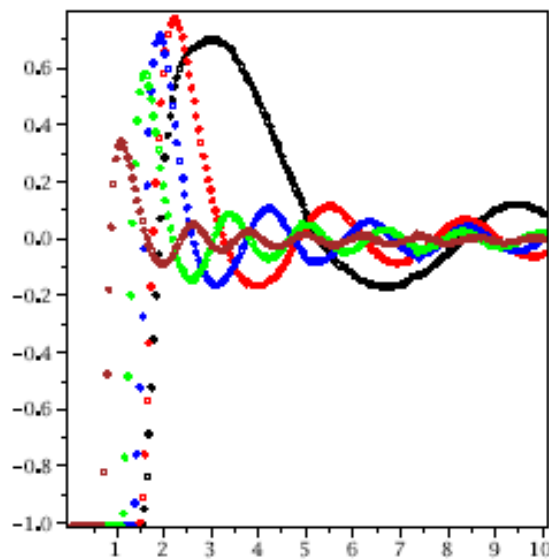
$$G^R(t, t', x) = \theta(t - t') \langle [O(t, x), O(t', 0)] \rangle$$

$$= G^R(t - t', x) = \int d\omega d^3k e^{-i\omega t + ikx} G^R(\omega, k)$$

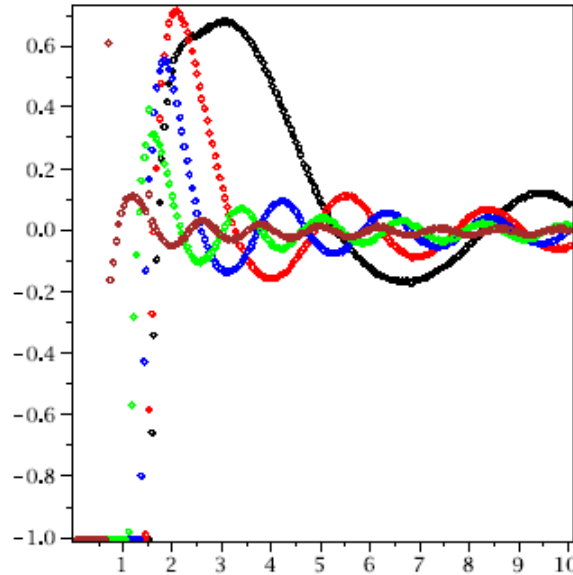
$$G^R(\omega, k) \rightarrow G_{thermal}^R(\omega, k)$$

Quasi-static approximation justified  
when  $\omega$  not too small, state not too  
close to equilibrium

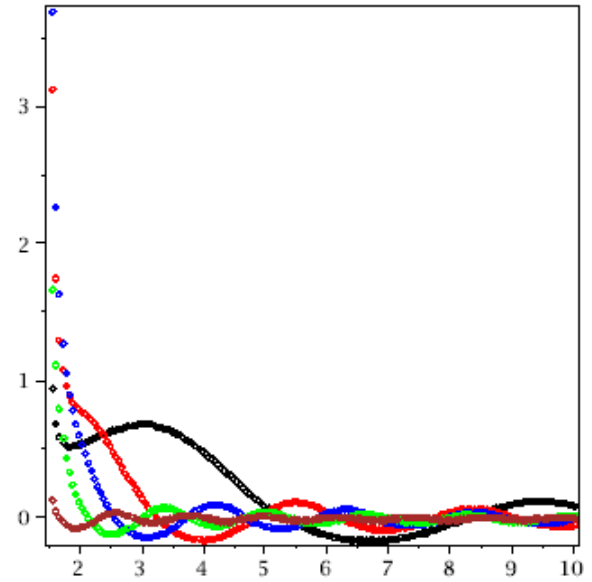
# Deviation of spectral function from thermal counterpart



$R_{xy}-\omega$ , stress



$R_{tx}-\omega$ , momentum density



$R_{tt}-\omega$ , energy density

$$\chi = -2 \text{Im} G^R(\omega)$$

$$R = \frac{\chi - \chi_{thermal}}{\chi_{thermal}}$$

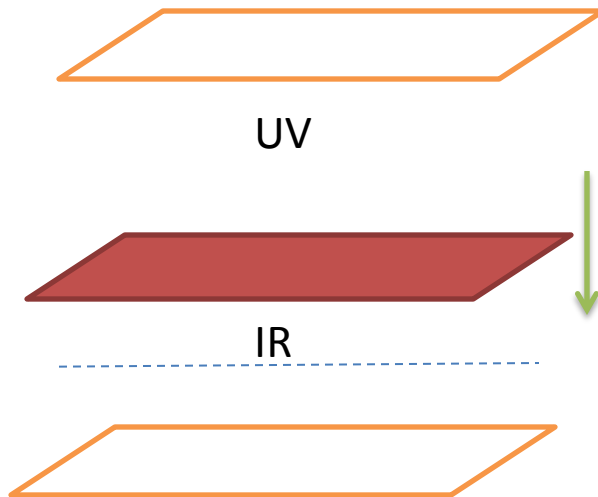
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Far from  
equilibrium

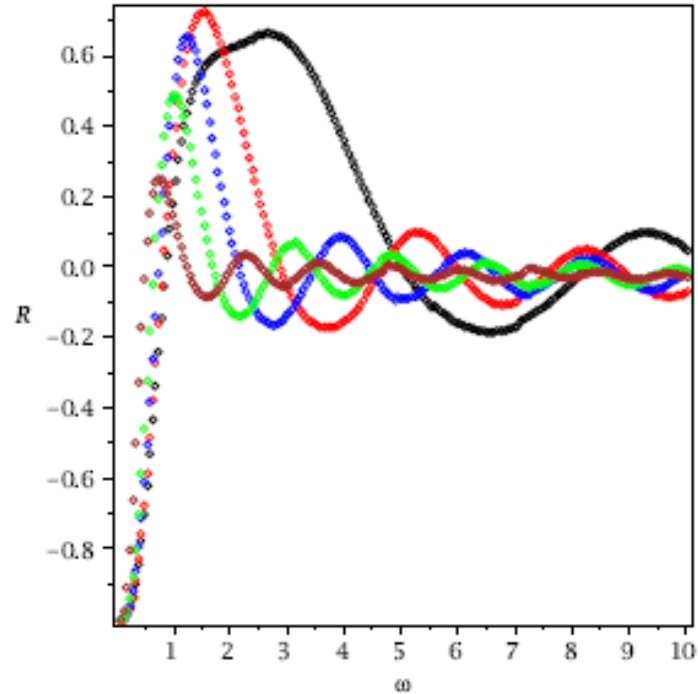
Near  
equilibrium

Deviation: general oscillations, increasing number of oscillations and shrinking amplitude.

# Top-down thermalization scenario



In contrast to weak coupling:  
bottom-up scenario  
(Baier, Mueller, Schiff, Son 11)



black   red   blue   green   brown

Far from equilibrium

Near equilibrium

$R \rightarrow 0$  for high frequency first.

Similar model, different probes: short distance scale  
thermalizes first (Balasubramanian et al 10)

# Correlator beyond quasi-static approximation

Consider dilaton for simplicity



AdS-Schwarzschild

$\phi_f(z, t_f, t')$

$$\phi_f(z \rightarrow 0, t) = \delta(t - t')$$

$$\square_f \phi_f = 0$$



matching: continuity of the dilaton and its flux across the shell



pure AdS

$\phi(z, t, t')$

$$\square \phi = 0$$



$\phi(z \rightarrow \infty, t)$  ingoing wave for retarded correlator

Erdmenger, SL JHEP 12

# PDE for massless bulk dilaton

$$G^R(t, t') = \int d^3x \theta(t - t') \langle [O(t, x), O(t', 0)] \rangle$$
$$\neq G^R(t - t')$$

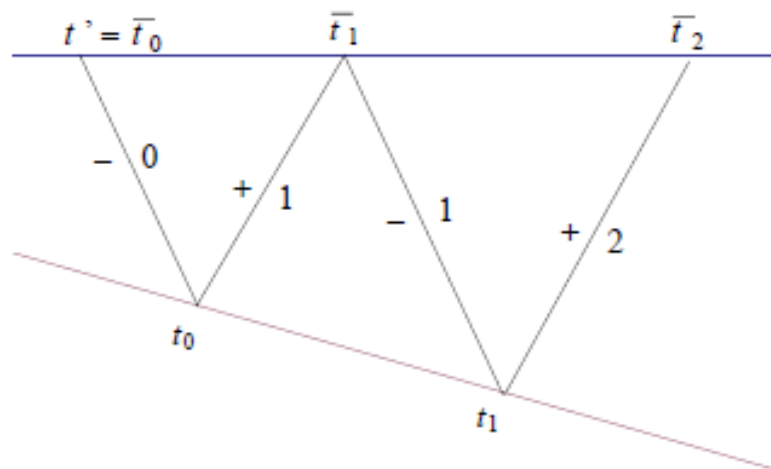
Retarded correlator for at  $q=0$  does not respect time translational symmetry

Need to solve PDE for  $\phi_f, \phi$  to obtain  $G^R(t, t')$ . Initial conditions required.

In the high frequency limit (WKB), waves follow geometric optics in AdS. PDE reduces to ODE, thus initial conditions not essential.

Instead of solving the PDE numerically, we use a geometric optics method.

# Illustration: divergence matching in pure AdS



Geometric optics in the bulk:

$\phi_f(t, t', z)$  singular along the segments  $(-,0)$ ,  $(+,1)$ ,  $(-,1)$  etc

positive/negative frequency contributions split in WKB limit

start with:

$(-,0)$   
 $\vdots$



singularities from bulk-boundary propagator in pure AdS

$(+,1)$ ,  $(-,1)$ ,  $(+,2)$ ...



singularities due to bouncing on the shell and the AdS boundary



# Illustration: divergence matching in pure AdS

Hologram: singularities in  $G^R(t, t')$ :  $G^R(t, t') = \int d^3x \theta(t - t') \langle [O(t, x), O(t', 0)] \rangle$

$$G^R(t \rightarrow \bar{t}_n, t') \sim \frac{B_n (-i)^{n-1}}{(-t + \bar{t}_n + i\varepsilon)^5} - \frac{B_n i^{n-1}}{(-t + \bar{t}_n - i\varepsilon)^5}$$

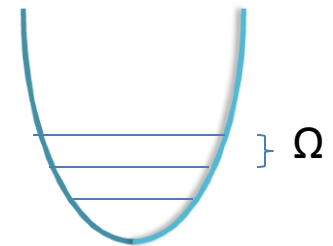
$n=0$  gives the light-cone singularity  $t' = \bar{t}_0$

$n>0$  singularities contain information on the spectrum of the glue ball operator: existence of equally spaced Normal Mode in the spectrum

Elementary example: nonrelativistic harmonic oscillator

$$K(x, t, x', 0) = \left[ \frac{m\Omega}{2i\pi \sin \Omega t} \right]^{1/2} \exp \left\{ \frac{im\Omega}{2 \sin \Omega t} [(x^2 + x'^2) \cos \Omega t - 2xx'] \right\}$$

Singularities at  $\Omega t = 2\pi k$

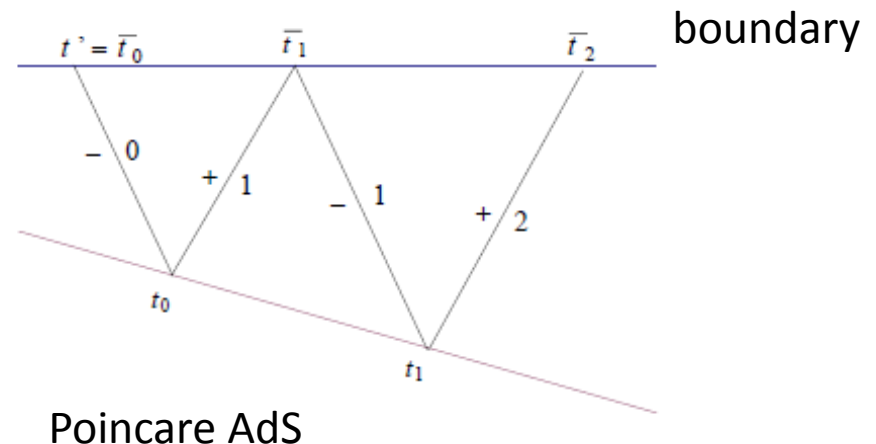
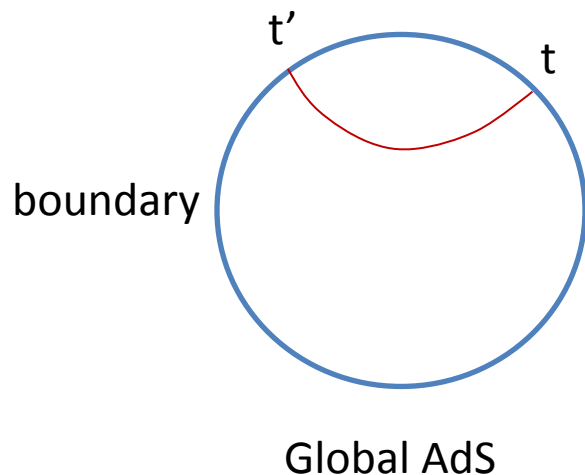


# Relation with bulk-cone singularities conjecture

$$G^R(t \rightarrow \bar{t}_n, t') \sim \frac{B_n (-i)^{n-1}}{(-t + \bar{t}_n + i\varepsilon)^5} - \frac{B_n i^{n-1}}{(-t + \bar{t}_n - i\varepsilon)^5}$$

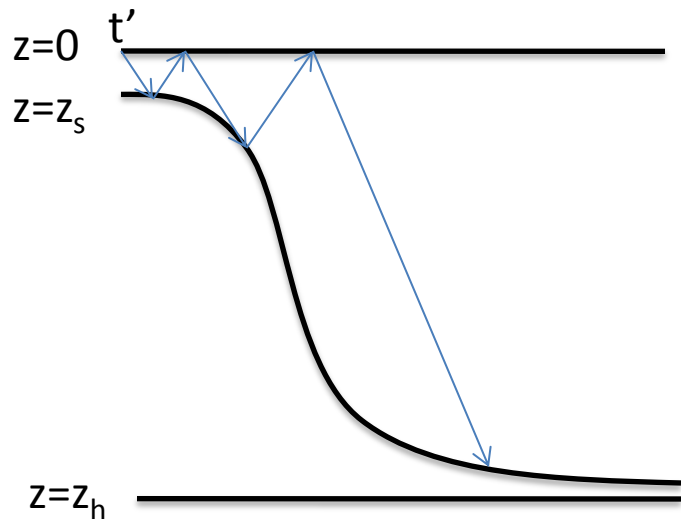
Bulk-cone singularities conjecture:  
Correlator becomes singular when  
two boundary points are connected  
by null geodesic in the bulk

Hubeny, Liu, Rangamani, JHEP 2007

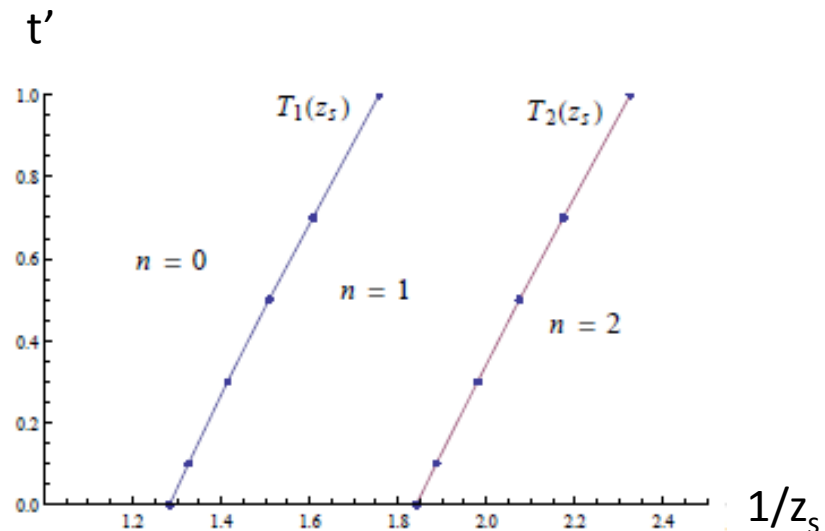


# Light ray bouncing in collapse background

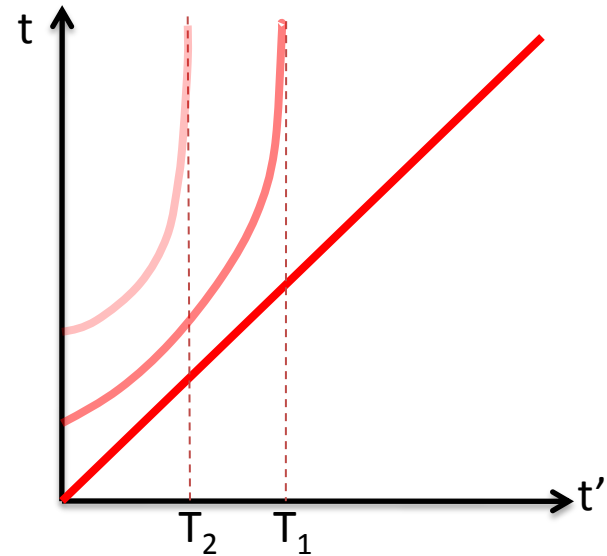
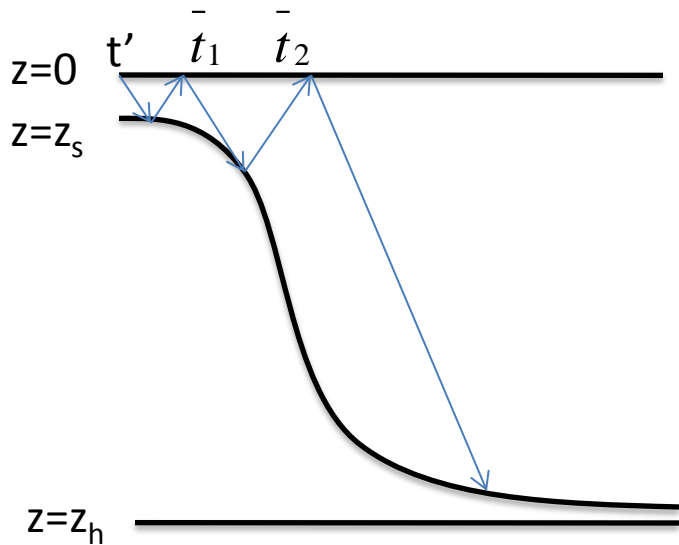
Expectation from geometric optics picture suggests singularities of  $G^R(t, t')$  when the light ray starting off at  $t'$  returns to the boundary



Only finite bouncings are possible:  
The warping factor freeze both the shell  
and the light ray near horizon



# Divergence matching in collapse background



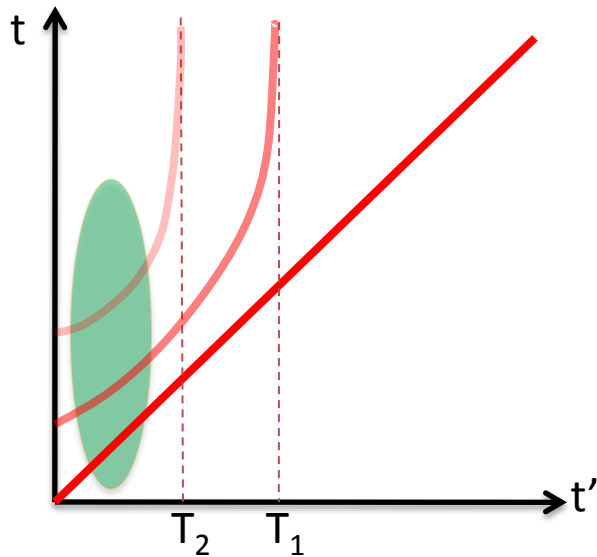
$$G^R(t \rightarrow \bar{t}_n) = \frac{A_n (-i)^{n-1}}{(-t + \bar{t}_n + i\varepsilon)^{5-n}} + \frac{A_n i^{n-1}}{(-t + \bar{t}_n - i\varepsilon)^{5-n}}$$

$$\bar{t}_n \rightarrow +\infty \quad \text{as} \quad t' \rightarrow T_n(z_s)$$

Singularities weakened after each bouncing

“thermalization time”  $t_{th} = \frac{T_1(\pi T z_s)}{\pi T} \sim \frac{O(1)}{\pi T}$

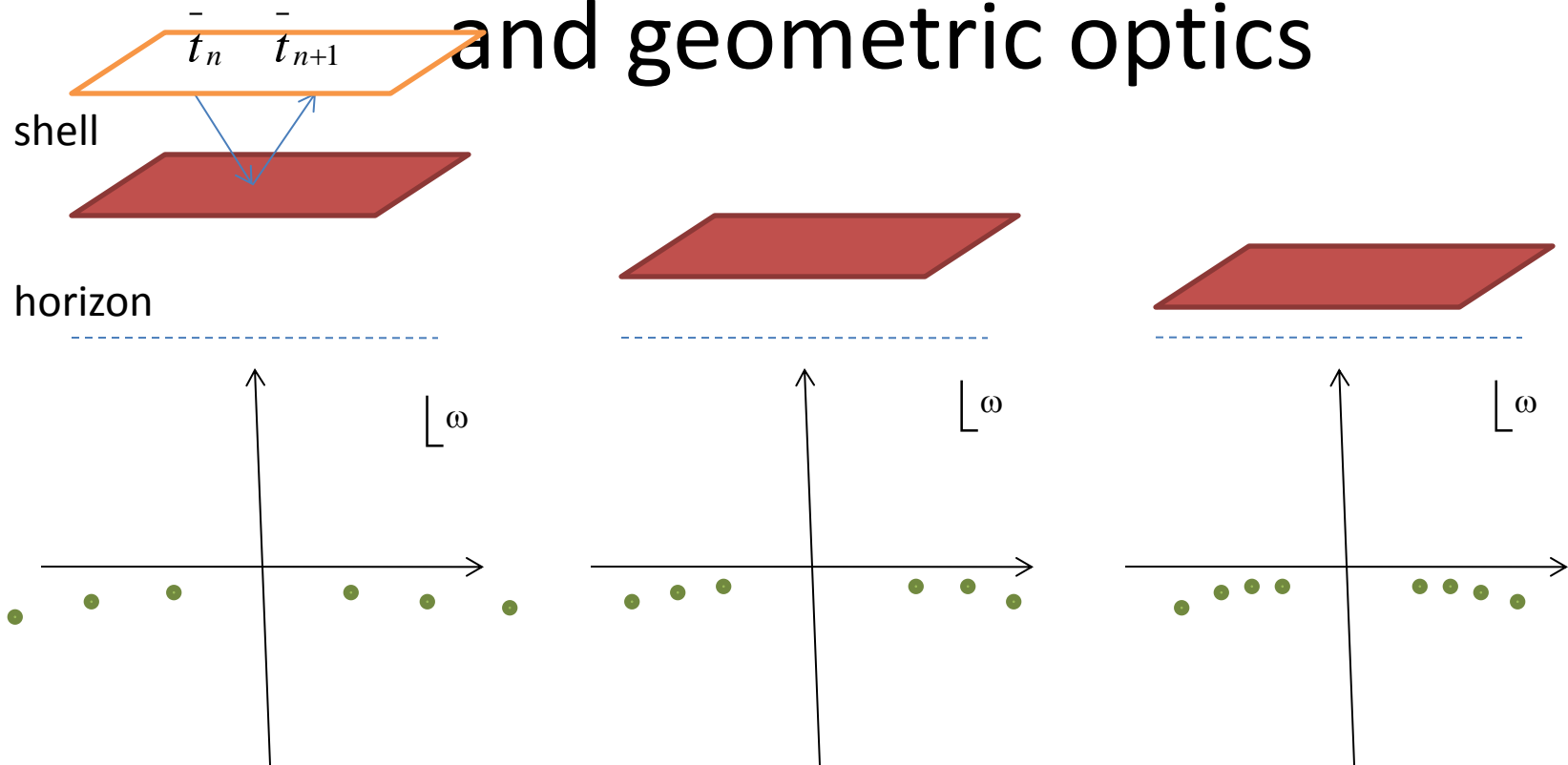
# Comparison between the two approximations



Green region: quasi-static approximation valid  
Red lines: geometric optics approximation valid  
Other region: needs solving PDE

$$G^R(t, t') = \int d^3x \theta(t - t') \langle [O(t, x), O(t', 0)] \rangle$$

# Overlap region for both quasi-static and geometric optics

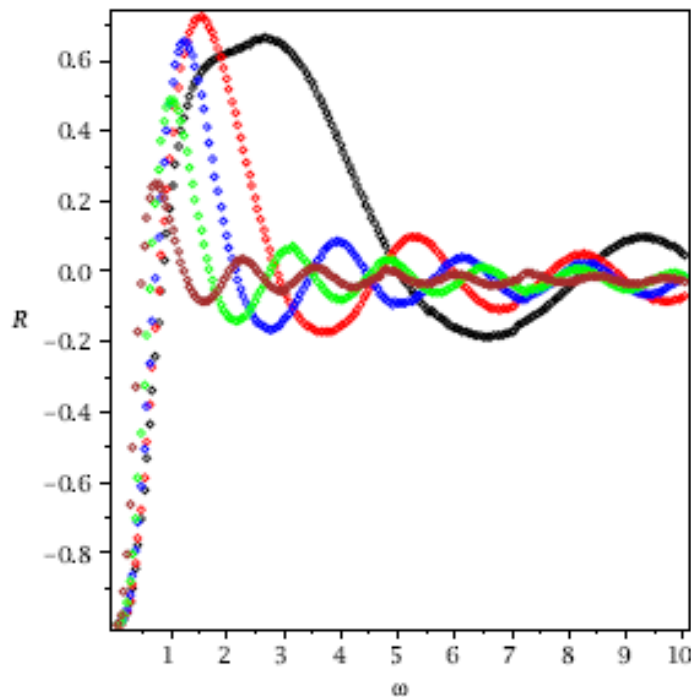


Approximate Normal Mode:  $\text{Re}(\omega) \gg \text{Im}(\omega)$

$$\Delta\omega \cdot (\bar{t}_{n+1} - \bar{t}_n) \approx 2\pi$$

See also Baier et al 1205.2998

# Deviation from thermal spectral function for quasi-static state



black    red    blue    green    brown

Far from  
equilibrium

Near  
equilibrium

$$\chi = -2 \operatorname{Im} G^R(\omega) \quad \text{Spectral function}$$

$$R = \frac{\chi - \chi_{\text{thermal}}}{\chi_{\text{thermal}}}$$

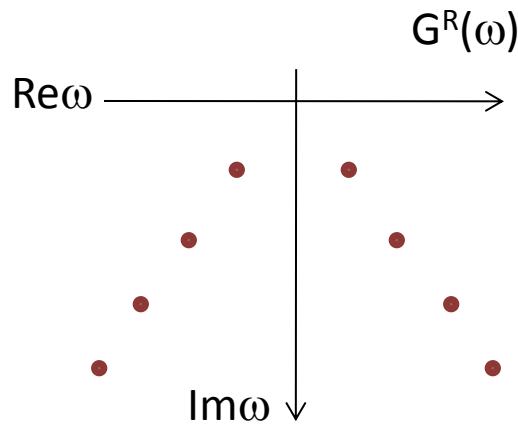
Spectral function for quasi-static state oscillate around thermal spectral function

Approximate Normal Mode responsible for the oscillation

$\Delta\omega$  shrinks as the shell is lowered toward the horizon

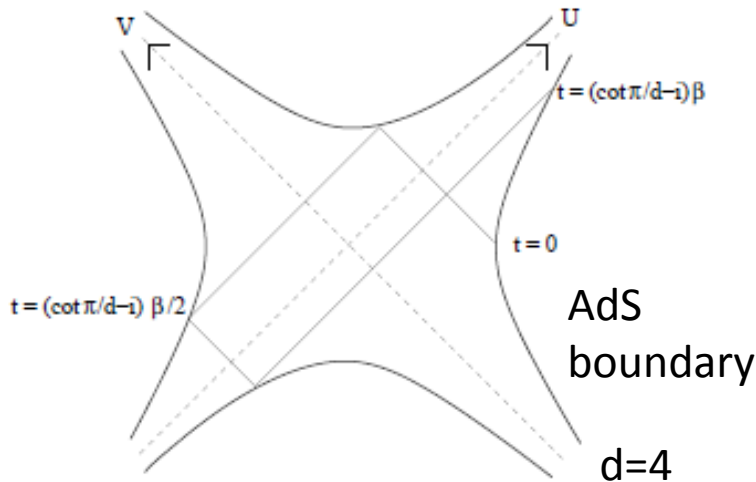
However, quasi-static approximation breaks down very close to equilibrium!

# Quasi-Normal Modes for thermal state



QNM for thermalized QGP obtained from scalar probe in AdS-Schwarzschild black hole

Quasi Normal Mode:  $\text{Re}(\omega) \sim \text{Im}(\omega)$



Bouncing light ray in Penrose diagram gives rise to complex time and therefore Quasi-Normal Modes

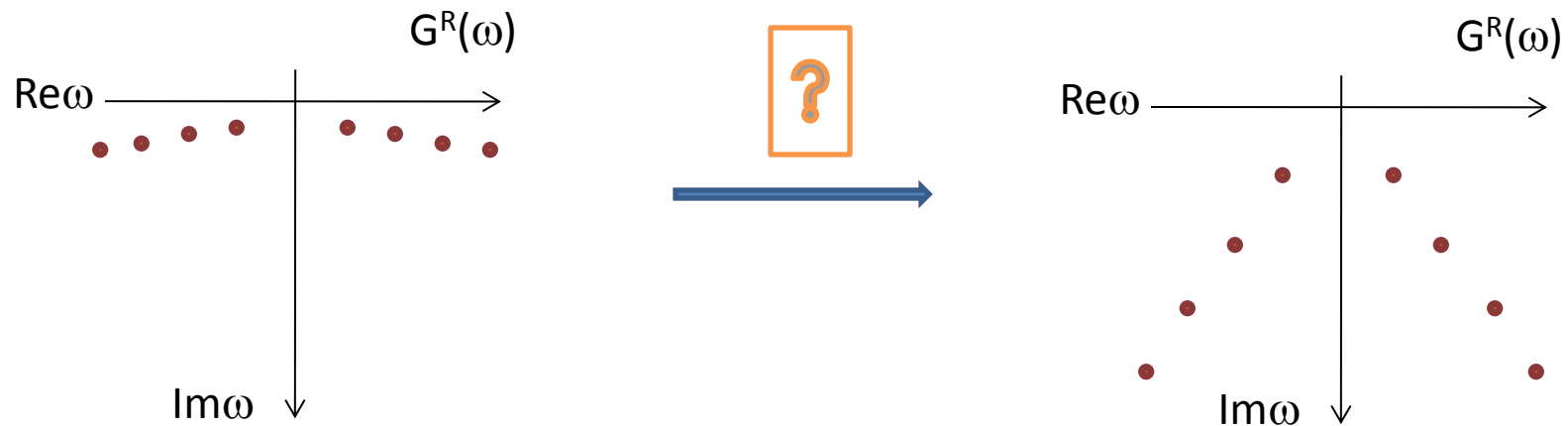
$$\Delta t = \frac{1}{2T} (1+i) \Rightarrow \Delta \omega = \frac{2\pi}{\Delta t} = 2\pi T (1-i)$$

$$\omega_n = 2\pi T (1-i)n$$

Amado & Hoyos JHEP 2008



# Evolution of singularities in thermalization



Quasi-static approximation breaks down.  
Need to go beyond the approximation to  
look at decay of low frequency mode.  
Outside reach of geometric optics (high  
frequency approximation).  
Need to solve full PDE

# Summary

- We studied falling of stringy debris produced in the gravity dual of heavy ion collisions.
- The hologram of separating quark-anti-quark pair shows no jet like structure.
- We constructed a gravitational collapse model for thermalization.
- We studied two point function in the thermalization history, using both quasi-static approximation and geometric optics approximation. Results consistent with a top-down thermalization scenario.
- In quasi-static approximation: we have established the the singularities as from the contribution of approximate Normal Modes, which are also responsible for the oscillation in spectral function.
- In geometric optics approximation: finite singularities; singularities weakened at late time, and eventually disappear at late stage of thermalization.
- Regions not covered by the two approximations need to be explored numerically.

Thank you!